Adaptive Meshless Methods for Solving Partial Differential Equations

KWOK Ting On

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Principal Supervisor: Dr. LING Leevan

Hong Kong Baptist University

September 2009
RBF collocation method is one of the most popular meshless computational method and many applications of RBF collocation method in different areas can be found. In this thesis, we discuss the RBF collocation method for solving partial differential equations and related issues. Since some non-stationary problems can be treated as stationary problems after discretizing the time derivative, we consider the stationary problems first. We give the convergence proof of a least-squares asymmetric radial basis function collocation method for solving the modified Helmholtz equations which is the reduced problem of some non-stationary problems such as heat equation. Second, we propose a method to enhance the performance for solving 3D inhomogeneous elliptic equations. The above methods show the excellent performance if the solution is smooth enough. Third, instead of uniform nodes, refinement nodes are used if the solution contains sharp region. We show that using refinement nodes results in more accurate RBF approximation than uniform nodes if the solution contains high variations. Besides, the Hybrid RBF approach will be discussed for meshless interpolation and approximation. Lastly, we combine all developed methods to give an adaptive method for solving time-dependent partial differential equations. Some numerical examples of the heat and the Burger’s equations will be given to conclude the work.
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