Generation of Entanglement and Its Decay in a Noisy Environment

HUANG Jiehui

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Principal Supervisor: Prof. ZHU Shiyao

Hong Kong Baptist University

December 2007
Abstract

Entanglement plays a central role in distinguishing quantum mechanics from classical physics. Due to its fantastic properties and many potential applications in quantum information science, entanglement is attracting more and more attention. This thesis focuses on the generation of entanglement and its decay in a noisy environment.

In the first experimental scheme to entangle two thermal fields, an atomic ensemble, composed of many identical four-level atoms, is employed. In the first Raman scattering, this atomic ensemble emits write signal photons after the pumping by a weak write pulse, accompanied by the transfer from one lower level to the other for some atoms. Similarly, the atomic ensemble emits read signal photons after the driving by a strong read pulse, and the ensemble turns back to its ground state after the second Raman scattering. The coherence between the two lower atomic levels plays a key role in establishing the quantum correlation between two emission fields, which is verified through the violation of Cauchy-Schwarz inequality. In particular, the controllable time delay between the two emission fields actually means the storage time of photonic information in this system, which sheds light on some potential applications, such as quantum memory.

In the second experimental scheme for the generation of spatially separated multi-photon entanglement, two or more identical optical cavities are aligned along a bee-line, and a four-level atom runs through these cavities sequentially. By appropriately adjusting the passage time of the atom in each cavity or the Rabi frequency of the classical pumping laser, a photon can be generated via the interaction between the excited atom and the cavity modes. This adiabatic passage model is an effective method to map atomic coherence to photonic state in cavity QED, thus all photons in different cavities quantum-mechanically correlate with the moving atom. When a final detection is made on this atom, a generalized n-photon GHZ entangled state will be generated with certainty.

Environment-induced disentanglement is another important topic in quantum optics. Based on the Peres-Horodecki criterion for separability of bipartite states, we develop the principal minor method for the verification of two-qubit entanglement.
Among the fifteen principal minors (seven effective ones) of a given two-qubit state’s partial transpose, if the minimum one is negative, the two-qubit state is entangled, otherwise it is separable. By applying this method to a two-qubit system under amplitude and phase dampings, we have derived the necessary and sufficient conditions for the entanglement sudden death of an initially entangled two-qubit state.

**Keywords:** entanglement generation, atomic ensemble, two-qubit, multiphoton entanglement, cavity QED, entanglement sudden death (ESD), amplitude damping, phase damping, principal minor.
Li-Zhi Liao (Chairman), Prof. Kok-Wai Cheah (as representative for Head of Department), Prof. Lei-Han Tang, Prof. Hang Zheng (EEx from Shanghai Jiao Tong University), and Prof. Xiang-Rong Wang (EEx from HKUST), for their efforts in reviewing and commenting on this work. Their feedback on my preliminary exam raised important questions that steered this work in the right direction.

Finally, I would like to thank my family for their support during my time at HKBU. To my parents who taught me the value of hard work, and sacrificed so much for my over 21 years’ education, I can’t thank you enough. This thesis is dedicated to you. I want to express my deep thanks to my mother and father in-law, especially to my wife Shuang-Zhu Zhou for putting up with me during the difficult time and keeping your patience and trust in me. Without your encouragement and love, I can not complete my PhD study or this thesis.
# Table of Contents

Declaration \hspace{1cm} i  

Abstract \hspace{1cm} ii  

Acknowledgements \hspace{1cm} iv  

Table of Contents \hspace{1cm} vi  

Chapter 1 \hspace{1cm} Backgrounds \hspace{1cm} 1  

1.1 Entanglement and Separability \hspace{1cm} 1  

1.2 Entanglement Measures \hspace{1cm} 3  

1.2.1 Two-qubit System \hspace{1cm} 4  

1.2.2 High Dimensional System \hspace{1cm} 12  

1.2.3 Multipartite System \hspace{1cm} 15  

1.3 Entanglement and Classical Correlation \hspace{1cm} 17  

1.4 Continuous Variable Entanglement \hspace{1cm} 20  

1.4.1 Continuous Variable Systems and Gaussian States \hspace{1cm} 20  

1.4.2 Descriptions of Continuous Variable States \hspace{1cm} 21  

1.4.3 Inseparability Criteria in Continuous Variable Systems \hspace{1cm} 25  

1.4.4 Gaussian Entanglement \hspace{1cm} 26  

1.5 Applications \hspace{1cm} 28  

1.5.1 Quantum Dense Coding \hspace{1cm} 28  

1.5.2 Quantum Teleportation \hspace{1cm} 29  

1.5.3 Quantum Cryptography \hspace{1cm} 30
Chapter 2   Entangling Two Thermal Fields by an Atomic Ensemble  
2.1 Introduction .............................................................. 33  
2.2 Dynamics in the Photon-atom System .............................. 34  
2.3 Normalized Second-order Correlation Function .................. 38  
2.4 Results and Discussions .............................................. 40  
2.4.1 Quantum Correlation of the Two Emission Fields ............. 40  
2.4.2 Effects of Decays .................................................... 43  
2.5 Summary ................................................................. 50  

Chapter 3   Generation of Multiphoton Entanglement in Cavity QED  
3.1 Introduction .............................................................. 52  
3.2 Proposed Experimental Setup ........................................ 54  
3.3 Emission of the First Photon ........................................ 55  
3.4 Emission of the Second Photon ...................................... 57  
3.5 Measurement Induced Two-photon Entanglement ................... 59  
3.6 Multiphoton Entanglement ............................................. 60  
3.7 Experimental Feasibility .............................................. 61  
3.8 Summary ................................................................. 62  

Chapter 4   Qubit Disentanglement in a Noisy Environment  
4.1 Introduction .............................................................. 63  
4.2 Decay of a Two-qubit System in a Noisy Environment ............ 64  
4.2.1 Amplitude Damping .................................................. 64  
4.2.2 Phase Damping ....................................................... 66  
4.3 Entanglement Sudden Death .......................................... 68  
4.4 Entanglement Verification Via Principal Minors ................... 71  
4.5 Necessary and Sufficient Condition for the ESD ................. 73  
4.5.1 Amplitude Damping .................................................. 73  
4.5.2 Phase Damping ....................................................... 79  
4.6 Time for the ESD ....................................................... 81  
4.7 Summary ................................................................. 82  

Chapter 5   Conclusions and Outlook  84
Appendix A  Supplementary Information to Chapter 2  88
  A.1  Spontaneous Emission in the Last Process of the First System . . . .  88
  A.2  Dynamics in the Third Process of the Second System . . . . . . . . .  95
  A.3  Explicit Forms of Eqs.(2.7), (2.12), and (2.19) . . . . . . . . . . . . .  101

Appendix B  Supplementary Information to Chapter 3  105
  B.1  Entangling the Moving Atom and the First Emitted Photon . . . .  105

Bibliography  109

Curriculum Vitae  122

List of Publications  123