On Two Tests for Multivariate Normality

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by

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Abstract

Assessing multivariate normality has long been an important topic in statistical inference. Many tests have been proposed. However, there is no widely accepted method. This thesis proposes two new tests for multivariate normality based on the density estimation, the sample entropy, the number-theoretic method and the projection pursuit technique. We extend the Vasicek (1976) test for univariate normality which is based on the sample entropy to the multivariate case. The problem of testing multivariate normality can be transformed into a problem of testing univariate marginal normality by the projection pursuit method. Since the projection pursuit method requires a set of projection directions with good uniformity, we develop an efficient method based on the number-theoretic method and the stochastic representation in distribution theory to generate the required set which is called an NT-net on $U_d$. We then propose two tests for multivariate normality considering a Kolmogorov-Smirnov type and a Cramer-von Mises type statistic. Their percentage points of the new statistics are obtained by the Monte Carlo simulation. The results of the power simulation show that the new tests are useful in practice. Two examples including the famous iris setosa data set illustrate our approach.
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