ABSTRACT

Image recovery and segmentation are always the fundamental tasks in image processing field, because of their so many contributions in practical applications. As in the past ten years, variational methods have achieved a great success on these two issues, in this thesis, we continue to work on proposing several new variational approaches for restoring and segmenting an image.

This thesis contains two parts. The first part addresses recovering an image and the second part emphasizes on segmenting. Along with the wide utilization of magnetic resonance imaging (MRI) technique, we particularly deal with blurry images corrupted by Rician noise. In chapter 1, two new convex variational models for recovering an image corrupted by Rician noise with blur are presented. These two models are motivated by the non-convex maximum-a-posteriori (MAP) model proposed in the prior papers. In the first method, we use an approximation item to the zero order of the modified Bessel function in the MAP model and add an entropy-like item to obtain a convex model. Through studying on the statistical properties of Rician noise, we bring up a strictly convex model by adding an additional data-fidelity term in the MAP model in the second method. Primal-dual methods are applied to solve the models. The simulation outcomes show that our models outperform some existed effective models in both recovery image quality and computational time.

Cone beam CT (CBCT) is routinely applied in image guided radiation therapy (IGRT) to help patient setup. Its imaging dose, however, is still a concern, limiting its wide applications. It has been an active research topic to develop novel technologies for radiation dose reduction. In chapter 2, we propose an improvement of practical CBCT dose control scheme - temporal non-local means (TNLM) scheme for IGRT. We denoise the scanned image with low dose by using the previous images as prior knowledge. We combine deformation image registration and TNLM. Different from the TNLM, in the new method, for each pixel, the search range is not fixed, but based on the motion vector between the prior image and the obtained image. By doing this, it is easy to find the similar pixels in the previous images, but also can reduce the computational time since it does not need large search windows. The phantom and patient studies illuminate that the new method outperforms the original one in both image quality and computational time.

In the second part, we present a two-stage method for segmenting an image corrupted by blur and Rician noise. The method is motivated by the two-stage segmentation method developed by the authors in 2013 and restoration method for images with Rician noise. First, based on the statistical properties of Rician noise, we present a new convex variant of the modified Mumford-Shah model to get the smooth cartoon part $u$ of the image. Then, we cluster the cartoon $u$ into different parts to obtain the final contour of different phases of the image. Moreover, $u$ from the first stage is unique because of the convexity of the new model, and it needs to be computed only once whenever the thresholds and the number of the phases $K$ in the second stage change. We implement the simulation on the synthetic and real images to show that our model outperforms some existed segmentation models in both precision and computational time.
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