Abstract

Two topics related to the experimental design are considered in this thesis. On the one hand, The uniform experimental design (UD), a major kind of space-filling design, is widely used in applications. The majority of UD tables (UDs) with good uniformity are generated under the centralized $L_2$-discrepancy (CD) and the wraparound $L_2$-discrepancy (WD). Recently, the mixture $L_2$-discrepancy (MD) is proposed and shown to be more reasonable than CD and WD in terms of uniformity. In first part of the thesis we review lower bounds for MD of two-level designs from a different point of view and provide a new lower bound. Following the same idea we obtain a lower bound for MD of three-level designs. Moreover, we construct UD}s under the measurement of MD by the threshold accepting (TA) algorithm, and finally we attach two new UD tables with good properties derived from TA under the measurement of MD. On the other hand, the problem of selecting a specific number of representative points (RPs) to maintain as much information as a given distribution has raised attention. Previously, a method has been given to select type-II representative points (RP-II) from normal distribution. These point sets have good properties and minimize the information loss. Whereafter, following similar idea, Fu, 1985 have discussed RP-II for gamma distribution. In second part of the thesis, we improve the discussion of selecting Gamma RP-II and provide more RP-II tables with a number of parameters. Further in statistical simulation, we also evaluate the estimation performance of point sets resampled from Gamma RP-II by making comparison in different situations.

Keywords: Hamming distance, Mixture discrepancy, Threshold accepting algorithm, Uniform design, Representative points, Loss function, Gamma distribution, Resampling, Moment estimation, Maximum likelihood estimation
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