Numerical Methods for Image Restoration

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Abstract

Numerical methods play an important role in seeking various efficient numerical solvers for a great deal of real world mathematical applications. In this thesis, we concentrate on some numerical study on the digital image restoration problem.

The aim of image restoration is to realize an estimate of the original image by making use of the information of the observed image. Total variation regularization method is a very popular regularization method in image processing due to its excellent ability to preserve edges in the recovered images. This regularization method is used in all restoration problems we have considered. In our work, we first consider the total variation denoising problem. Based on the theory of semismooth operators, we develop semismooth Newton's methods for total variation denoising algorithm. Then the restoration of blurred images corrupted by additive Gaussian noise is studied and a fast restoration method is proposed. An efficient model for the restoration of color images is also constructed. In addition, a blurred image corrupted by impulse noise plus Gaussian noise is considered and an effective two step method is established for its restoration. Multiplicative noise usually can distort an image seriously and almost all the information of the original image may disappear in the observed image, the removal of multiplicative noise is also studied and an effective globally convex model is proposed. Since different noises take different ways to affect an image, different noises removal in the above problems are greatly different. After that, by assuming that an image can belong to a Lipschitz space, the blind deconvolution problem is considered. In blind deconvolution study, the unknown blur and image can be recovered simultaneously. For all cases, the experimental results show that the proposed models are very efficient.
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