A Statistical Mechanical Study of Fat Tails
in Financial Time Series

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Abstract

Financial prices variations have been found to show some universal characteristics. Fat-tails, which are fatter than the gaussian tails, and short-time correlations play a key role in fundamental research as well as in risk management. In the first part, an empirical analysis on four years min-by-min Hang Seng Index have been performed to demonstrate the existence of these two features. Power-law fat-tail and slightly under-damping correlations were found. From the economic literature we already know that the underlying mechanism for the occurrence of fat-tails is the collective behavior of traders in a market. Fund managers, forecasts made by financial analysts and trust among traders all contribute to the "herding behavior".

In the second part, microscopic simulations are carried out on the original Cont-Bouchaud herding model to study the link of herding behavior of traders and the fat tails of price moves.

In the third part, a modified financial market herding model which uses the strategies of different traders to explain the short-time correlation is developed as well. We have been able to find a set of parameters which fit both the fat-tail and the short-time correlation of the Hong Kong financial market.
In the last part, a microscopic simulation approach has been applied to study the relations between microscopic behavior and macroscopic phenomena in financial markets. By changing the value of the parameters, some extreme cases are studied to explore the predictive power of the model for real markets.
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