Neural Networks for Optimization

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Abstract

Several neural network models are proposed in this thesis. These neural network models are established mainly based on the projection method. The projection method is well known for solving variational inequalities and constrained optimization problems in an iterative fashion. However, we only consider continuous-time neural network, which could be described by a set of ordinary differential equations. Firstly, we consider optimization problems with simple constraints. We have proved that our neural network model is well defined and the solution is unique. Also, the main result is the global convergence of the neural network and the equilibrium point is the corresponding solution of the optimization problem. Secondly, we apply the projection method to N-stage optimal control problems. The trajectory of the neural network for N-stage optimal control problems is unique and converges for any given initial point. The equilibrium point is the corresponding solution of the optimal control problem. At last, we focus on general constrained optimization problems. Two neural network models are introduced such that the equilibrium point of the corresponding dynamic system satisfy the Karush - Kuhn - Tucker (KKT) conditions. Numerical implementations showed the convergence of the projection method for neural network models. The neural network model for solving general constrained optimization problems also converge to the solution for most cases in our simulations.
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