Abstract

In this thesis, we investigate model checking problems for parametric single-index regression models when the variables are measured with different types of errors. The large sample behaviours of the test statistics can be used to develop properly centered and scaled model checking procedures. In addition, a dimension reduction model-adaptive strategy is employed, with the special requirements for the models with measurement errors, to improve the proposed testing procedures. This makes the test statistics converge to their weak limit under the null hypothesis with the convergence rates not depending on the dimension of predictor vector. Furthermore, the proposed tests behave like a classical local smoothing test with only one-dimensional predictor. Therefore the proposed methods have potential for alleviating the difficulties associated with high dimensionality in hypothesis testing.

Chapter 2 provides some tests for a parametric single-index regression model when predictors are measured with errors in an additive manner and validation dataset is available. The two proposed tests have consistency rates not depending on the dimension of predictor vector. One of these tests has a bias term that may become arbitrarily large with increasing sample size, but has smaller asymptotic variance. The other test is asymptotically unbiased with larger asymptotic variance. Both are still omnibus against general alternatives. Besides, a systematic study is conducted to give an insight on the effect of the ratio between the size of primary data and the size of validation data on the asymptotic behavior of these tests. Simulation studies are carried out to examine the finite-sample performances of the proposed tests. Also the tests are applied to a real data set about breast cancer with validation data obtained from a nutrition study.

Chapter 3 introduces a minimum projected-distance test for a parametric single-index regression model when predictors are measured with Berkson type errors. The distribution of the measurement error is assumed to be known up to several parameters. This test is constructed by combining the minimum distance test with a dimension reduction model-adaptive strategy. After properly centering, the minimum projected-distance test statistic is asymptotically normal at a convergence rate
of order $n h^{1/2}$ and can detect a sequence of local alternatives distinct from the null model with a rate of order $n^{-1/2} h^{-1/4}$ where $n$ is the sample size and $h$ is a sequence of bandwidths tending to 0 as $n$ tends infinity. These rates do not depend on the dimensionality of predictor vector, which implies that the proposed test has potential for alleviating the curse of dimensionality in hypothesis testing in this field. Further, as the test is asymptotically biased, two bias-correction methods are suggested to construct asymptotically unbiased tests. In addition, we discuss some details in the implementation of the proposed tests and then provide a simplified procedure. Simulations indicate desirable finite-sample performances of the tests. Besides, we illustrate the proposed model checking procedures by using two real datasets to illustrate the effects of air pollution on Emphysema.

Chapter 4 provides a nonparametric test for checking a parametric single-index regression model when predictor vector and response are measured with distortion errors. We estimate the true values of response and predictor, and then plug the estimated values into a test statistic to develop a model checking procedure. The dimension reduction model-adaptive strategy is also employed to improve its theoretical properties and finite sample performance. Another interesting observation in this work is that, with properly selected bandwidths and kernel functions in a limited range, the proposed test statistic has the same limiting distribution as that under the classical regression setup without distortion measurement errors. Simulation studies are conducted.

**Keywords:** Adaptive-to-Model Tests; Berkson Error; Bias Correction; Classical Error; Dimension Reduction; Distortion Error; Errors in Variables; Model Checking.
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